DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

319. Proposed by C. N. SCHMALL, New York City.

A man desires to purchase eggs at 5 cents, 1 cent, and $\frac{1}{2}$ cent, respectively, in such numbers that he will obtain 100 eggs for a dollar. How many solutions in rational integers?

I. Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.; J. W. CLAWSON, Ursinus College, Collegeville, Pa.; V. M. SPUNAR, Pittsburg, Pa.; and H. C. FEEMSTER, York College, York, Neb.

Let x=number at 5 cents, y=number at 1 cent, z=number at $\frac{1}{2}$ cent.
Then $x+y+z=100=5x+y+\frac{1}{2}z$...(1, 2).
Eliminating z we get the indeterminate equation, $9x+y=100$.
$\therefore y=100-9x$.
This equation gives us eleven integral solutions, as follows:
x=1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11.
y=91, 82, 73, 64, 55, 46, 37, 28, 19, 10, 1.
z=8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88.

Also solved by S. F. Norris, S. A. Corey, B. Kramer, J. Scheffer, T. J. Fitzpatrick, and Theodore L. DeLand.

II. Solution by PROFESSOR S. F. NORRIS, Baltimore City College, Baltimore, Md., and J. E. SANDERS, Weather Bureau, Chicago, Ill.

$$\text{Average price}=1\left\{\begin{matrix}5\\1\\\frac{1}{2}\end{matrix}\right\}\left|\begin{array}{r|r|r|r|r|r|r|r|r|r|r}1&2&3&4&5&6&7&8&9&10&11\\91&82&73&64&55&46&37&28&19&10&1\\8&16&24&32&40&48&56&64&72&80&88\end{array}\right|.$$

NOTE. This is the method used in many arithmetics. For the process of reasoning see *e. g.*, Ray's *New Higher Arithmetic*, p. 333, subject, Alligation. ED. F.

320. Proposed by FRANCIS RUST, C. E., Pittsburg, Pa.

Solve for t, $\cos t=m\cos 2t$.

Solution by T. J. FITZPATRICK, Lamoni, Iowa; S. A. COREY, Hiteman, Iowa; J. E. SANDERS, Weather Bureau, Chicago, Ill.; and B. KRAMER, E. M., N. S., Pittsburg, Pa.

$$\cos t=m\cos 2t=m(\cos^2t-\sin^2t)=m(\cos^2t-1+\cos^2)=2m\cos^2t-m.$$

$$2m\cos^2t-\cos t=m.$$

$$\cos^2t-\frac{1}{2m}\cos t=\tfrac{1}{2}.$$

$$\cos^2t-\frac{1}{2m}\cos t+\frac{1}{16m^2}=\tfrac{1}{2}+\frac{1}{16m^2}=\frac{8m^2+1}{16m^2}.$$

$$\cos t-\frac{1}{4m}=\pm\frac{1}{4m}\sqrt{[(8m^2+1)]}.$$

$$\cos t = \frac{1}{4m} \pm \frac{1}{4m}\surd[(8m^2+1)] = \frac{1}{4m}\{1 \pm \surd[(8m^2+1)]\}.$$

$$\therefore t = \cos^{-1}\{\frac{1}{4m}[1 \pm \surd(8m^2+1)]\}.$$

Also solved by G. B. M. Zerr, and H. C. Feemster.

GEOMETRY.

344. Proposed by C. N. SCHMALL, 604 East 5th Street, New York.

A tinsmith has a sheet of copper in the form of a rectangle, sides a and b. He desires to cut this into two pieces which will form a square when placed together. How can he do this?

I. Solution by V. M. SPUNAR, M. and E. E., East Pittsburg, Pa.

Assuming the base b of the rectangle to be divided into n and the other side a into $n-1$ equal parts. Then, cutting the whole figure by the broken line m, n, m', n', m'', n'', ..., pushing the left part one step down and shove it one step to the right, we will have for a square,

$$(n-1)x = ny.$$

Also, $a=(n-1)x$, and $b=ny$.

$$\therefore b : a = \left(\frac{n}{n-1}\right)^2.$$

As n must be a positive integer, the relation $b : a$ is *restricted* by the last equation. It is satisfied by the following series of numerical values.

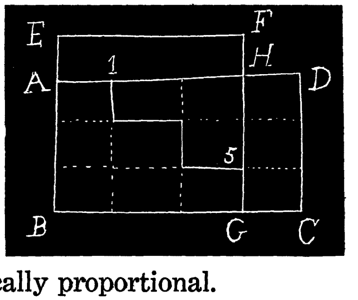
For $n=1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ..., \infty$.

$b : a = \infty, 4, 2\frac{1}{4}, 1\frac{7}{9}, 1\frac{9}{16}, 1\frac{11}{25}, 1\frac{13}{36}, 1\frac{15}{49}, 1\frac{17}{64}, 1\frac{19}{81}, ..., 1$.

Hence, the required division is possible when the sides of the rectangle are in the ratio 4 : 1.

Also solvee by S. Lefsehetz.

II. Solution by C. N. SCHMALL, 604 East 5th Street, New York.



Let $ABCD$ be the given rectangular sheet of copper; $AB=a$, $BC=b$. Let us suppose a square (of paper) $EBGF$ to be constructed by the usual method of the mean-proportional. Then the rectangles $AHFE$ and $GCDH$ are equivalent, and their sides are therefore reciprocally proportional.

$\therefore AH : HD = GH : HF$, or $BG : GC = AB : AE$.